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On B^0-B^0 Mixing and Violations of CP Symmetry

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ABSTRACT

In view of a possible long lifetime for B mesons $\tau_B \sim 10^{-12}$ sec - we re-examine predictions of the Standard Model for B^0-$\bar{B}^0$ mixing. We estimate theoretical uncertainties in the computation of the relevant matrix element by comparing the vacuum saturation approximation with predictions obtained in the bag model and in the harmonic oscillator model with and without relativistic corrections. For the B_d-$\bar{B}_d$ system we find mixing leading to a like- sign di-lepton yield of at most a few percent of the opposite sign di-lepton rate; B_s-$\bar{B}_s$ mixing should be stronger. While very little CP violation is expected in B^0-$\bar{B}^0$ mixing, we find that certain CP asymmetries in on-shell B_d decays could reach a level of 10% or more.

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I. MOTIVATION

For various reasons it is appropriate to re-analyze the prospects for B^0-$\bar{B}^0$ mixing and the observation of CP violation in B decays:

- The statistics that will be accumulated on B decays in the near future will allow B^0-$\bar{B}^0$ mixing and CP violations in these systems to be subjected to experimental scrutiny.
- The theoretical analysis of B^0-$\bar{B}^0$ mixing has so far employed arguments based largely on simplicity, like vacuum saturation of matrix elements, while ignoring other tools available in our theoretical arsenal to calculate matrix elements, like the bag model or the harmonic oscillator model.
- there are two new pieces of experimental information that are of direct relevance to such an analysis: an improved upper bound on the ratio of $b \to u$ over $b \to c$ transitions

$$\frac{\Gamma(b\to u)}{\Gamma(b \to c)} < 0.05 \qquad (1.1)$$

and a first tentative measurement of the B Meson lifetime $\tau(B) \sim 10^{-12}$ sec.[1]

In terms of the Kobayashi-Maskawa angles these numbers can conveniently be re-expressed as follows:[2]

$$s_2^2 + s_3^2 + 2 s_2 s_3 \cos\delta \simeq 4.2 \times 10^{-3}\, BR(b \to c)\left(\frac{10^{-12}\,\mathrm{sec}}{\tau_B}\right) \qquad (1.2)$$

$$s_3^2 \simeq 3.9 \times 10^{-2}\, BR(b \to u)\left(\frac{10^{-12}\,\mathrm{sec}}{\tau_B}\right) \qquad (1.3)$$

With $BR(b \to u) \lesssim 0.05$

and $\tau_B \sim 10^{-12}$ sec one finds $s_3 \lesssim 5\times10^{-2}$ (1.4)

Thus it is suggested that both s_2 and s_3 are considerably smaller than $s_1 \simeq 0.23$. If true it would lead to some interesting phenomenological consequences which will be discussed later.

The paper will be organized as follows: in sect. II we make a few short comments on the calculation of $\mathcal{L}_{eff}$, the effective interaction responsible for $B^0-\bar{B}^0$ mixing; in sec. III we discuss various methods to compute or at least estimate the matrix element $\langle B^0|\mathcal{L}_{eff}|\bar{B}^0\rangle$ and the amount of mixing;

sect. IV contains an analysis of how much CP violation might be expected in $B^0-\bar{B}^0$ mixing and in B decays; finally in sect. V we present our conclusions.

II CALCULATION OF $\mathcal{L}_{eff}$ ($\Delta B=2$)

The mass matrix determining $B^0-\bar{B}^0$ mixing is calculated by computing the well known box diagrams. Including strong radiative corrections one finds[3]

$$\mathcal{L}^{box}_{eff.} \simeq \eta_{QCD}\left(\frac{G_F}{4\pi}\right)^2\left\{\xi_t^2\left[m_t^2+\frac{1}{3}m_b^2+\frac{3}{4}m_b^2\log\frac{m_t^2}{m_b^2}\right]+O\left(m_c^2,\frac{m_b^4}{m_t^2}\right)\right\}\cdot$$

$$\cdot(\bar{b}q)_{V-A}(\bar{b}q)_{V-A} \qquad (2.1)$$

with $\xi_t = -s_1 s_2 e^{i\delta}$ in the small angle approximation and

$$\eta_{QCD} \simeq \left[\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right]^{-\frac{6}{23}}\left[\frac{3}{2}\left(\frac{\alpha_s(m_t)}{\alpha_s(M_W)}\right)^{-\frac{4}{7}}-\left(\frac{\alpha_s(m_t)}{\alpha_s(M_W)}\right)^{\frac{2}{7}}+\frac{1}{2}\left(\frac{\alpha_s(m_t)}{\alpha_s(M_W)}\right)^{\frac{8}{7}}\right]$$

b[q] stands for the bottom [down or strange] quark field. At short distances the effective $\Delta B=2$ coupling can be approximated by the box diagram. Such a procedure has first been applied to K^0-$\bar{K}^0$ mixing and it turned out to be rather successful in the sense that it lead to a correct prediction for the charm quark mass. However there are bound to be contributions to $\mathcal{L}_{eff}$ ($\Delta S=2$) which are <u>not</u> determined by short distance dynamics, namely K^0-$\bar{K}^0$ transitions that proceed via virtual $\pi, \eta, \pi\pi$ etc. intermediate states.(4) It is not known yet how to compute such contributions in a reliable fashion. We will come back to this complication later.

The B^0-$\bar{B}^0$ transition operator on the other hand should be determined basically by short distance dynamics, since M_B is so much heavier than typically hadronic mass scales; for the same reason one expects the spectator mechanism to yield a very good description of B decays. $\Gamma(b \to c)$ being much larger than $\Gamma(b \to u)$ strengthens the dominance of short distance dynamics even more since pionic intermediate states will be highly suppressed. Therefore

$$\mathcal{L}_{eff}(\Delta B=2) \simeq \mathcal{L}_{eff}^{box}(\Delta B=2) \tag{2.2}$$

III. ESTIMATES OF $\langle B^0 | \mathcal{L}_{eff}(\Delta B=2) | \bar{B}^0 \rangle$ and THE AMOUNT OF MIXING

The presumably gravest uncertainties arise when one tries to calculate the appropriate matrix element of $\mathcal{L}_{eff}$ since that is certainly not in the realm of short distance physics.

In our subsequent discussion we will use the following definition:

$$\mathcal{M} = \frac{1}{m_B} \langle \bar{B}^0 | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B^0 \rangle$$

$$= \frac{4}{3} R_B f_B^2 m_B \tag{3.1}$$

Saturating the matrix element (3.1) by inserting just the vacuum state yields $R_B=1$, where f_B is the decay constant of the B meson. This is the highly popular vacuum saturation approximation (=VSA).

The decay constant f_B can be measured in principle via the decay $B \to \tau \nu_\tau$, but at present it is not known. Theoretical estimates in the literature range between 100 MeV and 500 MeV. This uncertainty which is already quite unfortunate is further compounded by our ignorance concerning R_B, i. e. the numerical quality of the VSA.

It is tempting to use a value for R_B as extracted in the K^0-$\bar{K}^0$ system. There are however some drawbacks to such a procedure:

i. Since we do not know of a convincing argument for equating $\mathcal{L}_{eff}(\Delta S=2)$ with $\mathcal{L}_{eff}^{box}$ $(\Delta S=2)$, we cannot rely on $M(K_L)-M(K_S)$ for fixing R.

ii. Much better arguments can be given for CP violation to be determined by short distance dynamics. Yet to obtain ϵ_K one has to know the appropriate on-shell matrix element of the box operator for which a short distance analysis does not suffice. A priori there is no clear reason why R_K parameterizing this matrix element in K^0 physics should be equal to R_B.

iii. The interesting suggestion has been made to derive R from the observed $K^+ \to \pi^+ \pi^0$ width via current algebra [5] or chiral perturbation theory. [2] Such an argument is however not expected to work for the much heavier B meson - another reason why the parameter R_B could be quite different in the two cases.

Therefore we will estimate $\langle B^0 | \mathcal{L}_{eff} | \bar{B}^0 \rangle$ and its theoretical uncertainties by calculating it directly using phenomenological hadromic wavefunctions. Of these wavefunctions we demand of course that they give at least a decent fit to non-strange and strange hadrons. We will actually employ three different models each stressing different aspects and thus hopefully complementing each other:

(A) The non-realistic harmonic oscillator model = HO

(B) The relativistically corrected HO model = RHO

(C) The bag model = BM

Although no clear reason can be given why the HO and the RHO model should work in the first place, one does not expect them to give a poorer description of bottom states than of strange states. As far as a bag model is concerned where hadrons are taken as a sphere it has even be suggested to provide a more reliable description for mesons containing one - but not two - heavy quarks. [7,8]

Our discussion of case (A) and (B) parallels the treatment of K^0-$\bar{K}^0$ mixing in ref. 6.

A. The HO Model

The harmonic oscillator quark model of Isgur and Karl[9] ignores all relativistic effects and therefore has to be considered as a phenomenological ansatz, however as a amazingly successful one.

The matrix element (eq. (3. 1)) is found to be given by

$$\mathcal{M}^{HO}_{(b\bar{q})} = 16\left(\frac{\beta^2_{b\bar{q}}}{\pi}\right)^{3/2} \qquad (3.2)$$

$$\beta^2_{b\bar{q}} = \left(\frac{2 m_b m_q \varkappa}{m_b + m_q}\right)^{1/2} \qquad (3.3)$$

where $\varkappa$ describes the coupling strength of the HO potential. Fitting the model yields[9] $\varkappa \sim 10^{-2} (\mathrm{GeV})^3$. With the (constituent) quark masses M_d=1/3 GeV, M_s=0.5 GeV and M_b=5 GeV one obtains

$$\mathcal{M}^{HO}_{B_d} \simeq 6.4 \cdot 10^{-2} \, (\mathrm{GeV})^3 \qquad (3.4)$$

$$\mathcal{M}^{HO}_{B_s} \simeq 8.5 \cdot 10^{-2} \, (\mathrm{GeV})^3 \qquad (3.5)$$

B. RHO Model

Colic et al. include some relativistic corrections to the HO model by replacing Pauli spinors by Dirac spinors.

They find

$$\mathcal{M}_B^{RHO} = 16\,\hat{a}\left[1 - \frac{6\beta_B^2}{(E_b+m_b)(E_q+m_q)} + \frac{9\beta_B^4}{(E_b+m_b)^2(E_q+m_q)^2}\right] \tag{3.6}$$

with

$$\hat{a} = \alpha_B\left(\frac{\beta_B^2}{\pi}\right)^{3/2}, \quad \alpha_B = \frac{1}{4E_bE_q}(E_b+m_b)(E_q+m_q)$$
$$E_i = \left(m_i^2 + \tfrac{3}{2}\beta_B^2\right)^{1/2} \tag{3.7}$$

Inserting the same mass values as before we obtain:

$$\mathcal{M}_{B_d}^{RHO} \simeq 4.8 \cdot 10^{-2}\ (\mathrm{GeV})^3 \tag{3.8}$$

$$\mathcal{M}_{B_s}^{RHO} \simeq 7.2 \cdot 10^{-2}\ (\mathrm{GeV})^3 \tag{3.9}$$

C. The bag model

The bag model is complimentary to the HO model in the sense that it allows for a fully relativistic description. There one can derive the following results:[10]

$$\mathcal{M}_B^{bag} = 8(a - b - 4c) \tag{3.10}$$

where

$$a = \frac{N_b^2N_q^2}{2\pi}\int_0^R dr\, r^2\left[j_0^2\left(\frac{\xi_q r}{R}\right)j_0^2\left(\frac{\xi_b r}{R}\right) + \epsilon_b^2\epsilon_q^2 j_1^2\left(\frac{\xi_q r}{R}\right)j_1^2\left(\frac{\xi_b r}{R}\right)\right] \tag{3.11}$$

$$b = \frac{N_b^2N_q^2}{2\pi}\int_0^R dr\, r^2\left[\epsilon_b^2 j_0^2\left(\frac{\xi_q r}{R}\right)j_1^2\left(\frac{\xi_b r}{R}\right) + \epsilon_q^2 j_0^2\left(\frac{\xi_b r}{R}\right)j_1^2\left(\frac{\xi_q r}{R}\right)\right] \tag{3.12}$$

$$c = \frac{N_b^2N_q^2}{2\pi}\int_0^R dr\, r^2\,\epsilon_b\epsilon_q\, j_0\left(\frac{\xi_q r}{R}\right)j_1\left(\frac{\xi_q r}{R}\right)j_0\left(\frac{\xi_b r}{R}\right)j_1\left(\frac{\xi_b r}{R}\right) \tag{3.13}$$

with the normalization constants

$$N_i^{-2} = \int_0^R dr\, r^2 \left[j_0^2\left(\frac{\xi_i r}{R}\right) + \epsilon_i^2 j_1^2\left(\frac{\xi_i r}{R}\right) \right] \tag{3.14}$$

R=radius of the hadron and the kinematical factors

$$\epsilon_i^2 = \frac{(\xi_i^2 + m_i^2 R^2)^{1/2} - m_i R}{(\xi_i^2 + m_i^2 R^2)^{1/2} + m_i R} \tag{3.15}$$

The j_i are spherical Bessel functions and the ξ_i represent momenta in units of the natural scale R^{-1}-$p_i=\xi_i R^{-1}$- and are determined to be roots of the equation

$$tg\, \xi_i = \frac{\xi_i}{1 - m_i R - (\xi_i^2 + m_i^2 R^2)^{1/2}} \tag{3.16}$$

For d quarks with $M_d \simeq 0$ one finds $\xi_d \simeq 2.043_1$ while s quarks with $M_s \sim 280$ MeV imply $\xi_s \simeq 2.43$; b quarks on the other hand with $M_b \sim 4.6$-4.8 GeV are so heavy that they can safely be treated non-relativistically. [8]

This limit is obtained by letting $m_b R$ go to infinity; then

$\xi_b = \pi$ and simplifications occur (e. g. ,c=0 since $\epsilon_b=0$). We obtain

$$\mathcal{M}_{B_d}^{bag} \simeq 3.9 \cdot R^{-3} \simeq 6 \times 10^{-2} (GeV)^3 \tag{3.17}$$

$$\mathcal{M}_{B_s}^{bag} \simeq 5.5 \times R^{-3} \simeq 8.5 \times 10^{-2} (GeV)^3 \tag{3.18}$$

with $R^{-1} \simeq 025$ GeV [8] with $R^{-1} \simeq 0.25$ GeV[8].

Instead of $m_b R \to \infty$ one can use $m_b \sim 5$ Gev in which case $\xi_b \simeq 3.064$. Then one finds

$$\mathcal{M}_{B_d}^{bag} \simeq 5.2 \times 10^{-2}\,(GeV)^3$$

$$\mathcal{M}_{B_s}^{bag} \simeq 7.7 \times 10^{-2}\,(GeV)^3$$

i. e. very little difference from (3. 17,3. 18). This illustrates our statement that the bag model results are not very sensitive to the choice of the b quark mass

In table I we have summarized our results obtained so far and have them juxtaposed to the vacuum saturation approximation VSA.

$\mathcal{M}_B$ $[\times 10^{-2}\,(GeV)^3]$	VSA	HO	RHO	bag
B_d	$15.8 \times \left[\frac{f_B}{150\,MeV}\right]^2$	6.4	5.1	5.2
B_s	$15.8 \times \left[\frac{f_B}{150\,MeV}\right]^2$	8.5	7.2	7.7

TABLE I

Some comments are in order:

i. The three models, in particular the harmonic oscillator model and the bag model, yield very similar numerical results although the light anti-quark is treated in a very different fashion, namely relativistically in the bag model and non-relativistically in the HO model. This agreement does not hold for the K^0-$\bar{K}^0$ case.

ii. The value f_B=150 MeV was picked somewhat arbitrarily to illustrate the order of magnitude of the VSA result. Potential models yield $f_{B_d} \simeq 125 [f_{B_s} \simeq 175]$MeV while a bag model calculation [8] gives $f_{B_d} \simeq 100$ MeV. Thus one estimates

$$\mathcal{M}_{B_d}^{VSA} \simeq (7 \div 11) \times 10^{-2} (GeV)^3 \, , \; R_{B_d} \simeq 0.5 - 0.8$$

iii. The bag model result is fairly stable under variation of parameters and positive in sign. This is in marked contrast to the K^0-$\bar{K}^0$ case where small variations in the bag parameters affect the magnitude of $\mathcal{M}_K$ drastically and can even change the sign [6]. For completeness it should be kept in mind that the <u>spherical</u> bag model being used here will cease to offer a reasonable description when the anti-quark becomes too heavy.

iv. One should recall that in non-relativistic models the decay constant f_M of a meson with mass M is given by the wavefuction at the origin

$$f_M^2 = \frac{12 |\psi(0)|^2}{M} \tag{3.19}$$

and therefore

$$\mathcal{M} = \frac{4}{3} R_B f_M^2 M = 16 R_B |\psi(0)|^2 \tag{3.20}$$

Thus a non-relativistic treatment suggests that $\mathcal{M}$ has the same value for all mesons with the same reduced mass μ and increases with μ - as borne out by comparing $\mathcal{M}_k^{HO}$ and $\mathcal{M}_B^{HO}$.

To summarize our analysis so far; using VSA for computing the matrix element $\mathcal{M}_B$ with $f_B \simeq 150$ MeV might lead to an overestimate by a factor of roughly two.

The amount of mixing is usually expressed in terms of

$$x = \frac{2\Delta m}{\Gamma} \tag{3.21}$$

where

$$\Delta m = m_1 - m_2 = 2 \,\mathrm{Re}\, M_{12} \tag{3.22}$$

$$M_{12} = \left(\frac{G_F}{4\pi}\right)^2 \mathcal{M}\,\eta_{QCD} \left\{ \xi_t^2 \left(m_t^2 + \frac{1}{3} m_b^2 + \frac{3}{4} m_b^2 \log \frac{m_t^2}{m_b^2}\right) + O\left(m_c^2 \, m_b^4 / m_t^2\right) \right\} \tag{3.23}$$

For $B_d \equiv (b\bar{d})$ mesons we then find with $\xi_t^2 \simeq s_1^2 s_2^2 e^{2i\delta}$

$$x_{B_d} \simeq 13\, s_2^2 \cos 2\delta \times \left[\frac{\mathcal{M}}{5\times 10^{-2} (\mathrm{GeV}^3)}\right] \left[\frac{\tau_B}{10^{-12}\,\mathrm{sec}}\right] \tag{3.24}$$

where we have set $m_t \sim 35$ GeV. To a first approximation x_{B_d} scales like $(m_t/35\ \mathrm{GeV})^2$. In ref. 2 it was shown that for $\tau_B \sim 10^{-12}$ sec, $m_t \sim 35$GeV (and $R_K \sim 0.37$) one can derive a lower limit on $|s_2 s_3 \sin\delta|$ from the measured value of ε_K and $M(K_L)-M(K_S)$:

$$|s_2 s_3 \sin\delta| \gtrsim 2.2\times 10^{-3} \quad \text{if} \quad \cos\delta < 0 \tag{3.25}$$

Even accepting all these values does not suffice yet to fix all the relevant parameters; however upper bounds can be given like $s_2 \lesssim 0.1$ from eq. (1.2,4) and cos 2 $\delta \lesssim 2/3$ from eq. (3. 25). Assuming these upper bounds to be saturated we find as an order of magnitude estimate

$$x_{B_d} \simeq 0.13 \times \left[\frac{\mathcal{M}}{5\times 10^{-2} (\mathrm{GeV})^3}\right] \tag{3.26}$$

Thus VSA yields $x_{B_d} \sim 0.4$ while the bag model ansatz leads to $x_{B_d} \sim 0.14$.

For $B_s=(b\bar{s})$ mesons we find much larger values.
Since $\xi_t^2\simeq(s_2+s_3e^{i\delta})^2e^{2i\delta}$ in that case we get

$$x_{B_s} \sim 260\times F \times \left[\frac{\mathcal{M}}{5\times10^{-2}(\mathrm{GeV})^3}\right]\left[\frac{\tau_B}{10^{-12}\,\mathrm{sec}}\right] \tag{3.27}$$

$$F = \left[s_2^2\cos 2\delta + 2s_2s_3\cos\delta + s_3^2\right] \tag{3.28}$$

For $\tau_B\sim10^{-12}$ sec one can write

$$F \simeq \left[4.2\times10^{-3} - 2s_2^2\sin^2\delta\right] \tag{3.29}$$

Again no firm prediction of F and thus of x_{B_s} can be given at present. Saturating the lower bound on $|s_2\sin\delta|$ derived from eq. (1. 4,3. 25) one finds $F\sim10^{-3}$ and therefore

$$x_{B_s} \sim 0.3\times\left[\frac{\mathcal{M}}{5\times10^{-2}(\mathrm{GeV})^3}\right] \tag{3.30}$$

Thus VSA gives $x_{B_s}\sim1$ while the bag model yields $x_{B_s}\sim0.5$.

B^0-$\bar{B}^0$ mixing will lead to like-sign dileptons in e^+e^-

annihilation.Since the $B^0\bar{B}^0$ pair is produced in a coherent quantum state one has to include effects due to Bose statistics. If $B^0\bar{B}^0$ are produced in a p wave state-as it happens on $\Upsilon(4s)$-one finds for the dileptons from $B^0\bar{B}^0$ decays

$$R_{\ell=1} = \frac{N(\ell^+\ell^+)+N(\ell^-\ell^-)}{N(\ell^+\ell^-)} = \frac{x^2}{2+x^2} \tag{3.31}$$

With the numbers give above we find

$$R_{\ell=1}(B_d) \simeq 0.01 - 0.08 \tag{3.32}$$

$$R_{\ell=1}(B_s) \simeq 0.10 - 0.30 \tag{3.33}$$

For ℓ(= relative orbital angular momentum) = even as in $e^+e^- \to B^*\bar{B} \to B\bar{B}\gamma$ one obtains much larger values:

$$R_{\ell=\text{even}}(B_d) = \frac{3x^2 + x^4}{2 + x^2 + x^4} \simeq 0.04 - 0.23 \tag{3.34}$$

$$R_{\ell=\text{even}}(B_s) \simeq 0.30 - 1 \tag{3.35}$$

To stress it again: these numbers are not firm predictions, since the relevant parameters are not sufficiently well known yet; they are given to illustrate the order of magnitude of such effects and their inherent theoretical uncertainties.

IV. CP VIOLATION IN B MESON TRANSITIONS

It has to be kept in mind that CP violation can surface in B meson transition in two different ways:

a. It can occur in B^0-$\bar{B}^0$ mass mixing as it occurred in K^0-$\bar{K}^0$ mixing where it is characterized by the quantity ε_K.

b. It can appear also in on-shell B decays in analogy to the quantity ε' defined in K decays.

i. It has been pointed out before[3] that chances to observe CP violation in B^0-$\bar{B}^0$ mixing are rather slim. The relevant parameters are defined as follows:

$$r \equiv \frac{\Gamma(B^0 \to \ell^+ \nu X)}{\Gamma(B^0 \to \ell^- \bar{\nu} X)} = \left|\frac{1-\epsilon_B}{1+\epsilon_B}\right|^2 \frac{(\Delta m)^2 + \frac{1}{4}(\Delta\Gamma)^2}{2\Gamma^2 + (\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2} \qquad (4.1)$$

$$\bar{r} \equiv \frac{\Gamma(\bar{B}^0 \to \ell^- \bar{\nu} X)}{\Gamma(\bar{B}^0 \to \ell^+ \nu X)} = \left|\frac{1+\epsilon_B}{1-\epsilon_B}\right|^2 \frac{(\Delta m)^2 + \frac{1}{4}(\Delta\Gamma)^2}{2\Gamma^2 + (\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2} \qquad (4.2)$$

With these quantities one can express the total lepton charge asymmetry in semi-leptonic decays of the $B^0\bar{B}^0$ system:[3]

$$A_\ell \equiv \frac{N(\ell^+) - N(\ell^-)}{N(\ell^+) + N(\ell^-)} = \frac{r - \bar{r}}{2 + r + \bar{r}} \qquad (4.3)$$

It is a general feature of the standard six quark model that A_1 is very small independent of the values of s_2,s_3 and m_t: $A_1 < 10^{-2}$.

Small values of s_2 and s_3 and $m_t \gtrsim 30$ Gev will actually decrease A_1 much further: $A_1 \sim 10^{-4} - 10^{-3}$.

ii. From this pessimistic estimate of CP violation in B^0-$\bar{B}^0$ mixing one should not infer that CP violation will be unobservable in B decays. The pattern of CP violation in the B^0 system could well be completely different from the one in the K^0 system: on-shell B decays could show sizeable CP asymmetries as explained in ref. 12,13. Here we will re-analyze those predictions.

The main idea is that there are certain final states f into which both B^0 and $\bar{B}^0$ can decay - possibly after a multistep reaction:

$$\begin{matrix} B^0 \searrow \\ \bar{B}^0 \nearrow \end{matrix} f$$

Candidates for such a final state are:

$$\begin{array}{l} B_d \to D^0 + \pi' s \searrow \\ \qquad\qquad\qquad\qquad K_s + \pi' s \\ \bar{B}_d \to \bar{D}^0 + \pi' s \nearrow \end{array}$$

$$\begin{array}{l} B_d \searrow \\ \qquad D\bar{D}K_s + \pi' s \\ \bar{B}_d \nearrow \end{array}$$

Mixing is evoked to provide interference of two amplitudes thus exposing possible complex phases in them; however CP violation in the mixing itself is <u>not</u> required. One can define a CP asymmetry in $e^+e^- \to B^0\bar{B}^0 + X$:

$$A = \frac{\sigma(\ell^+\bar{X}f) - \sigma(\ell^-Xf)}{\sigma(\ell^+\bar{X}f) + \sigma(\ell^-Xf)} = -\frac{2xa^2 \sin 2\phi}{1+y^2+y\cos 2\phi} \tag{4.4}$$

if $B^0\bar{B}^0$ are produced in a charge conjugation even state, like $e^+e^- \to \bar{B}^*\bar{B} \to B^0\bar{B}^0\gamma$; it vanishes otherwise. Here we have used the notation:

$$x = \frac{2\Delta m}{\Gamma}, \quad y = \frac{\Delta\Gamma}{\Gamma}, \quad a = \frac{1-y^2}{1+x^2}$$

$$-e^{-2i\phi} = \frac{pM}{q\bar{M}}, \quad \frac{p}{q} = \frac{1+\epsilon_B}{1-\epsilon_B}$$

$$M = \langle f|\mathcal{L}(\Delta B{=}1)|B^0\rangle, \quad \bar{M} = \langle f|\mathcal{L}(\Delta B{=}1)|\bar{B}^0\rangle \tag{4.5}$$

The cases of B_d and B_s decays have to be treated separately; first we consider the $B_d\bar{B}_d$ system:

$$\sin 2\phi_d = \frac{s_3^2 \sin 2\delta + 2s_2 s_3 \sin\delta}{s_2^2 + s_3^2 + 2s_2 s_3 \cos\delta}$$

$$= \frac{2s_3 \sin\delta\,(s_3\cos\delta + s_2)}{s_2^2 + s_3^2 + 2s_2 s_3 \cos\delta} \qquad (4.6)$$

It is again premature to give a firm prediction for $\sin 2\phi_d$, but using "typical" values for s_2, s_3 and δ that satisfy the relations (1. 2,1. 4,3. 25) one finds as reasonable estimate $\sin 2\phi_d \sim 1/3$ and therefore

$$|A_d| \sim 11-30\% \qquad (4.7)$$

Of course it will be very hard to analyze special exclusive decay modes and still accumulate sufficient statistics. Instead one can compare inclusive modes like $B^0B^0 \to \ell^- K_S X$ vs. $B^0B^0 \to \ell^+ K_S X$ or $B^0B^0 \to \ell^+\ell^-_D X$ vs. $B^0B^0 \to \ell^+\ell^-_D X$ where the second lepton comes from D decays. When performing such a semi-inclusive analysis one will certainly include channels with no CP asymmetry and thus dilute the effect. However in view of the possible size of A_d(eq. (4. 7)) this is a viable option.

The prospects are much more gloomy for the $B_s\bar{B}_s$ system since one finds within the standard six quark model:

$$\sin 2\phi_s = 0 + O\left(\left(\frac{m_c}{m_t}\right)^2, s_i^2\right) \tag{4.8}$$

V. SUMMARY

The neutral B mesons offer a great opportunity to study complex phenomena like B^0-$\bar{B}^0$ mixing and CP violation.

A long B meson lifetime $\sim 10^{-12}$ sec will certainly increase the observability of such effects. Numerical predictions for the relevant quantities are hampered by uncertainties in the size of certain matrix elements. We estimated these uncertainties to amount to factor of roughly three. While we consider these model predictions to be more reliable for B^0-$\bar{B}^0$ than for K^0-$\bar{K}^0$ matrix elements we do not see a convincing argument for extrapolating experimental information on the K^0-$\bar{K}^0$ system to the B^0-$\bar{B}^0$ system.

As far as mixing is concerned, we expect some mixing to occur for B_d-$\bar{B}_d$ leading to a like-sign dilepton rate of at most a few percent of the opposite-sign dilepton yield. For B_s-$\bar{B}_s$ on the other hand we except very sizeable mixing.

CP violation in B^0-$\bar{B}^0$ mixing will be very hard to observe if the standard six quark model is correct. Very sizeable CP asymmetries might however appear in on-shell B_d decays. This could be studied in e^+e^- annihilation just above the $B\bar{B}^*$ threshold or on the Z^0 resonance.

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REFERENCES

1. S. Stone, Invited lecture at the Lepton Photon Symposium, Cornell University (1983).
2. P. Ginsparg ad M. B. Wise, preprint HUTP-83/A027 (1983).
3. J. S. Hagelin, Nucl. Phys. B193 (1981) 123, and references therein; E. Franco et al, Nucl. B194 (1982) 403.
4. L. Wolfenstein Nucl. Phys. B160 (1979) 501.
5. J. Donoghue, E. Golowich and B. Holstein, Phys.Lett. 119B (1982) 412.
6. P. Colic et al., preprint MPI-PAE/P Th 39/82 (1982).
7. W. A. Ponce, Phys. Rev. D19 (1979) 2197.
8. E. Golowich, Phys. Lett. 91B (1980) 271.
9. N. Isgur and G. Karl, Phys. Lett. 74B (1978) 353; Phys. Rev. D18 (1978)4187, ibid. D20 (1979)1191.
10. R. E. Shrock and S. B. Treiman, Phys. Rev. D19 (1979)2148; B. McWilliams and O. Shankar, ibid. D22 (1980)2853. Our sign differs from that of these authors and is in agreement with that of ref. 6. To support our statement we mention that the sign of the VSA result should agree with the axial-axial contribution in the bag model.
11. See for example: H. Kraseman, Phys Lett. 96B (1980)397.
12. I. I. Bigi and A. I. Sanda, Nucl. Phys. B193 (1981)85.
13. A. B. Carter and A. I. Sanda, Phys. Rev. D23 (1981)1567.